

بسم الله الرحمن الرحيم

King Abdulaziz University



Solution of The Second-Final Exams

Hamed Al-Sulami

- أنقر على Start لبدء الاختبار.
- يحتوي هذا الاختبار على عشرون سؤالاً.
- عند الانتهاء من الاختبار أنقر على End للحصول على النتيجة.
- بالتوفيق إن شاء الله.

Calculus II
Math202

© 2013
April 24, 2013

hhaalsalmi@kau.edu.sa
Version 1.0



Enter Name:

I.D. Number:

Answer each of the following.

1. $\int \sin^2 x \, dx =$

$$\frac{x}{2} - \frac{\sin(2x)}{4} + C$$

$$\frac{x}{2} + \frac{\sin(2x)}{4} + C$$

$$\frac{x}{2} - \frac{\cos(2x)}{4} + C$$

$$\frac{x}{2} + \frac{\cos(2x)}{4} + C$$

$$\begin{aligned} 2. \int \cos(5x) \cos(3x) dx &= \\ &= -\frac{\sin(2x)}{4} + \frac{\sin(8x)}{16} + C \\ &= \frac{\sin(2x)}{2} + \frac{\sin(8x)}{8} + C \\ &= \frac{\sin(2x)}{4} + \frac{\sin(8x)}{16} + C \\ &= -\frac{\sin(2x)}{4} - \frac{\sin(8x)}{16} + C \end{aligned}$$

$$3. \int \tan^3 x \sec^3 x dx =$$

$$\frac{\sec^5 x}{5} + \frac{\sec^3 x}{3} + C$$

$$\frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$$

$$-\frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$$

$$-\frac{\sec^5 x}{5} + \frac{\sec^3 x}{3} + C$$

$$4. \int \frac{\sqrt{4-x^2}}{x^2} dx =$$

$$-\frac{\sqrt{4-x^2}}{x} + \sin^{-1}\left(\frac{x}{2}\right) + C$$

$$\frac{\sqrt{4-x^2}}{x} - \sin^{-1}\left(\frac{x}{2}\right) + C$$

$$\frac{\sqrt{4-x^2}}{x} + \sin^{-1}\left(\frac{x}{2}\right) + C$$

$$-\frac{\sqrt{4-x^2}}{x} - \sin^{-1}\left(\frac{x}{2}\right) + C$$

5. $\int \frac{1}{\sqrt{x^2-1}} dx =$

$$\ln |x + \sqrt{x^2 - 1}| + C$$

$$\ln |x - \sqrt{x^2 - 1}| + C$$

$$\sin^{-1} x + C$$

$$\cos^{-1} x + C$$

$$\begin{aligned} 6. \int \frac{x}{\sqrt{x^2 + 49}} dx &= \\ \frac{\sqrt{x^2+49}}{7} + C & \\ \frac{\sqrt{x^2+49}}{14} + C & \\ \sqrt{x^2 + 49} + C & \\ -\frac{\sqrt{x^2+49}}{x} - \tan^{-1}\left(\frac{x}{7}\right) + C & \end{aligned}$$

$$\begin{aligned} 7. \int \frac{1}{(x+4)(x+5)} dx &= \\ \ln \left| \frac{x+4}{x+5} \right| + C & \\ -\ln \left| \frac{x+4}{x+5} \right| + C & \\ \ln \left| \frac{x-4}{x+5} \right| + C & \\ \ln \left| \frac{x+4}{x-5} \right| + C & \end{aligned}$$

$$8. \int \frac{1}{x^2 \sqrt{4-x^2}} dx =$$

$$\frac{\sqrt{4-x^2}}{x} + C$$

$$-\frac{\sqrt{4-x^2}}{x} + C$$

$$\frac{\sqrt{4-x^2}}{4x} + C$$

$$-\frac{\sqrt{4-x^2}}{4x} + C$$

$$9. \int \frac{1}{\sqrt{8-x^2-2x}} dx =$$

$$-\sin^{-1} \left(\frac{x+1}{3} \right) + C$$

$$\sin^{-1} \left(\frac{x+1}{3} \right) + C$$

$$\cos^{-1} \left(\frac{x+1}{3} \right) + C$$

$$\sin^{-1} \left(\frac{x+1}{2} \right) + C$$

10. $\int \frac{1}{x^2(x+3)} dx =$

$$\frac{1}{9} \ln \left| \frac{x+3}{x} \right| + \frac{1}{3x} + C$$

$$\frac{1}{9} \ln \left| \frac{x}{x+3} \right| - \frac{1}{3x} + C$$

$$-\frac{1}{9} \ln \left| \frac{x+3}{x} \right| + \frac{1}{3x} + C$$

$$\frac{1}{9} \ln \left| \frac{x+3}{x} \right| - \frac{1}{3x} + C$$

11. $\int \frac{1+\sin x}{\sin^2 x} dx =$

$$\cot x + \ln |\csc x - \cot x| + C$$

$$-\cot x + \ln |\csc x - \cot x| + C$$

$$-\cot x - \ln |\csc x - \cot x| + C$$

$$\cot x + \ln |\csc x + \cot x| + C$$

12. $\int \frac{\sqrt{x}}{1+x^3} dx =$

$$\tan^{-1}(x^3) + C$$

$$\frac{2}{3} \tan^{-1}(x^{3/2}) + C$$

$$\frac{2}{3} \sin^{-1}(x^{3/2}) + C$$

$$\tan^{-1}(x^{3/2}) + C$$

13. $\int \frac{e^{2x}}{1+e^x} dx =$

$e^x - \ln(e^x - 1) + C$

$e^x + \ln(e^x + 1) + C$

$e^x - \ln(e^x + 1) + C$

$e^{-x} - \ln(e^x + 1) + C$

14. $\int_1^{\infty} \frac{8}{x^5} dx =$

1

2

$\frac{8}{5}$

Divergent

15. $\int \frac{1}{\sqrt{x^2 + 2x + 5}} dx =$

$\ln(x + 1 - \sqrt{x^2 + 2x + 5}) + C$

$\ln(x - 1 - \sqrt{x^2 + 2x + 5}) + C$

$\ln(x - 1 + \sqrt{x^2 + 2x + 5}) + C$

$\ln(x + 1 + \sqrt{x^2 + 2x + 5}) + C$

16. $\int \frac{x+1}{x^2-7x+12} dx =$

$\ln \left(\frac{|x+4|^5}{(x-3)^4} \right) + C$

$\ln \left(\frac{|x-4|^5}{(x-3)^4} \right) + C$

$\ln \left(\frac{|x-4|^5}{(x+3)^4} \right) + C$

$\ln \left(\frac{|x+4|^5}{(x+3)^4} \right) + C$

17. $\int \frac{x^3 + x}{x^2 - 1} dx =$

$$\frac{1}{2}x^2 + \ln|x^2 - 1| + C$$

$$\frac{1}{2}x^2 - \ln|x^2 - 1| + C$$

$$x^2 + \frac{1}{2} \ln|x^2 - 1| + C$$

$$\frac{1}{2}x^2 - \frac{1}{2} \ln|x^2 - 1| + C$$

18. The partial decomposition of $\frac{4x+6}{(x^2+1)(x-1)^2}$ is

$$\frac{3x-2}{x^2+1} + \frac{3}{x-1}$$

$$\frac{3x-2}{x^2+1} + \frac{5}{(x-1)^2}$$

$$\frac{3x-2}{x^2+1} + \frac{-3}{x-1} + \frac{5}{(x-1)^2}$$

$$\frac{-3x-2}{x^2+1} + \frac{3}{x-1} + \frac{5}{(x-1)^2}$$

19. The integral $\int_0^{\infty} \frac{1}{e^x + e^{-x}} dx$

Converges to $\frac{\pi}{4}$

Converges to $\frac{\pi}{2}$

Converges to $\frac{3\pi}{4}$

Diverges

20. The integral $\int_0^1 x \ln x dx$

Diverges

Converges to 0

Converges to $\frac{1}{4}$

Converges to $-\frac{1}{4}$

21. The integral $\int_1^e \frac{1}{x \sqrt[3]{\ln x}} dx$

$\frac{3}{2}$

$\frac{-3}{2}$

$\frac{2}{3}$

Diverges

$$\begin{aligned} 22. \int \frac{3x^2 - x + 5}{x(x^2 + 5)} dx &= \\ \frac{3x - 2}{x^2 + 1} + \frac{3}{x - 1} & \\ \frac{3x - 2}{x^2 + 1} + \frac{5}{(x - 1)^2} & \\ \frac{3x - 2}{x^2 + 1} + \frac{-3}{x - 1} + \frac{5}{(x - 1)^2} & \\ \frac{-3x - 2}{x^2 + 1} + \frac{3}{x - 1} + \frac{5}{(x - 1)^2} & \end{aligned}$$

$$23. \int \frac{1}{x + \sqrt[3]{x}} dx =$$

$$\tan^{-1}(x^3) + C$$

$$\frac{2}{3} \tan^{-1}(x^{3/2}) + C$$

$$\frac{2}{3} \sin^{-1}(x^{3/2}) + C$$

$$\tan^{-1}(x^{3/2}) + C$$

$$24. \int \tan x \sec^3 x dx =$$

$$\frac{\sec^3 x}{4} + C$$

$$\frac{\sec^3 x}{3} + C$$

$$\frac{\tan^4 x}{4} + C$$

$$\frac{\sec^3 x}{3} + \sec x + C$$

25. $\int \frac{1}{\sqrt{x^2+4}} dx =$

$$\ln \left| \frac{x}{2} + \frac{\sqrt{x^2+4}}{2} \right| + C$$

$$\ln \left| \frac{x}{2} - \frac{\sqrt{x^2+4}}{2} \right| + C$$

$$\sin^{-1} \left(\frac{x}{2} \right) + C$$

$$\cos^{-1} \left(\frac{x}{2} \right) + C$$

26. $\int \frac{x^3+8}{x+2} dx =$

$$\frac{x^3}{3} + x^2 + 4x + C$$

$$\frac{x^3}{3} - x^2 + 4x + C$$

$$\frac{x^3}{3} - x^2 - 4x + C$$

$$\frac{x^3}{3} + x^2 - 4x + C$$

$$27. \int (\cos^4 x - \sin^4 x) dx =$$

$$x + \frac{\sin(2x)}{2} + C$$

$$\sin x \cos x + C$$

$$x + \frac{\cos(2x)}{2} + C$$

$$\frac{\cos^5 x}{5} - \frac{\sin^5 x}{5} + C$$

$$28. \int x\sqrt{x+1} dx =$$

$$\frac{2}{5}(x+1)^{5/2} + \frac{2}{3}(x+1)^{3/2} + C$$

$$\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$$

$$\frac{2}{5}(x+1)^{5/2} - \frac{1}{3}(x+1)^{3/2} + C$$

$$\frac{1}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$$

29. $\int \frac{e^x}{(e^x+2)(e^x+3)} dx =$

$\ln(e^x + 2) + \ln(e^x + 3) + C$

$\ln(e^x + 2) - \ln(e^x + 3) + C$

$\ln(e^x + 3) - \ln(e^x + 2) + C$

$\ln(e^x - 2) - \ln(e^x - 3) + C$

30. The integral $\int_1^{\infty} 2xe^{-x^2} dx$

Converges to $\frac{1}{e}$

Converges to $-\frac{1}{e}$

Converges to $\frac{2}{e}$

Diverges

31. $\int \sin(5) \cos(7x) dx =$

$$-\frac{\sin(2x)}{4} + \frac{\sin(12x)}{16} + C$$
$$\frac{\cos(5) \sin(7x)}{7} + C$$
$$\frac{\sin(5) \sin(7x)}{7} + C$$
$$-\frac{\sin(2x)}{2} - \frac{\sin(12x)}{12} + C$$

Answers:

Points:

Percent:

Letter Grade:

Solutions to Quizzes

Solution to 1.

$$\begin{aligned} & \int \sin^2 x \, dx \\ &= \int \frac{1}{2} [1 - \cos(2x)] \, dx \qquad \sin^2 x = \frac{1 - \cos(2x)}{2} \\ &= \frac{1}{2} \left[x - \frac{1}{2} \sin(2x) \right] + C \qquad \int \cos(ax) = \frac{1}{a} \sin(ax) + C \\ &= \frac{x}{2} - \frac{\sin(2x)}{4} + C. \end{aligned}$$



Solution to 2.

$$\int \cos(5x) \cos(3x) dx$$

$$= \int \frac{1}{2} [\cos(5x - 2x) + \cos(5x + 3x)] dx \quad \cos A \cos B = \frac{\cos(A - B) + \cos(A + B)}{2}$$

$$= \frac{1}{2} \int [\cos(2x) + \cos(8x)] dx \quad \int \cos(ax) = \frac{1}{a} \sin(ax) + C$$

$$= \frac{1}{2} \left[\frac{1}{2} \sin(2x) + \frac{1}{8} \sin(8x) \right] + C$$

$$= \frac{\sin(2x)}{4} + \frac{\sin(8x)}{16} + C.$$



Solution to 3.

$$\int \tan^3 x \sec^3 x \, dx$$

$$= \int \tan^2 x \sec^2 x \sec x \tan x \, dx \qquad \tan^2 x = \sec^2 x - 1$$

$$= \int (\sec^2 x - 1) \sec^2 x \sec x \tan x \, dx \qquad u = \sec x, \, du = \sec x \tan x \, dx$$

$$= \int (u^2 - 1)u^2 \, du$$

$$= \int u^4 - u^2 \, du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C \qquad u = \sec x$$

$$= \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C.$$

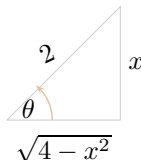


Solution to 4. Let

$$x = 2 \sin \theta,$$

$$dx = 2 \cos \theta d\theta$$

$$\sqrt{4 - x^2} = 2 \cos \theta.$$



$$\int \frac{\sqrt{4 - x^2}}{x^2} dx$$

$$= \int \frac{2 \cos \theta}{4 \sin^2 \theta} 2 \cos \theta d\theta$$

$$= \int \frac{4 \cos^2 \theta}{4 \sin^2 \theta} d\theta$$

$$= \int \cot^2 \theta d\theta$$

$$= \int (\csc^2 \theta - 1) d\theta$$

$$= -\cot \theta - \theta + C = -\frac{\sqrt{4 - x^2}}{x} - \sin^{-1} \left(\frac{x}{2} \right) + C.$$



Solution to 5.

Using the fact

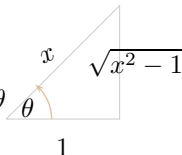
$$\begin{aligned}\int \frac{1}{\sqrt{x^2 - a^2}} dx &= \ln \left| \frac{x}{a} + \frac{\sqrt{x^2 - a^2}}{a} \right| + C \\ &= \ln |x + \sqrt{x^2 - a^2}| + C \\ \int \frac{1}{\sqrt{x^2 - 1}} dx &= \ln |x + \sqrt{x^2 - 1}| + C.\end{aligned}$$

Another solution: Let

$$x = \sec \theta,$$

$$dx = \sec \theta \tan \theta d\theta$$

$$\sqrt{x^2 - 1} = \tan \theta.$$



$$\int \frac{1}{\sqrt{x^2 - 1}} dx$$

$$= \int \frac{1}{\tan \theta} \sec \theta \tan \theta d\theta$$

$$= \int \sec \theta d\theta$$

$$= \ln |\sec \theta + \tan \theta| + C$$

$$= \ln |x + \sqrt{x^2 - 1}| + C.$$



Solution to 6.

$$\begin{aligned}& \int \frac{x}{\sqrt{x^2 + 49}} dx \\&= \frac{1}{2} \int (x^2 + 49)^{-1/2} 2x dx \quad \int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + C \\&= \frac{1}{2} \frac{(x^2 + 49)^{1/2}}{\frac{1}{2}} + C \\&= \sqrt{x^2 + 49} + C.\end{aligned}$$



Solution to 7.

$$\frac{1}{(x+4)(x+5)} = \frac{A}{x+4} + \frac{B}{x+5}$$

$$1 = A(x+5) + B(x+4)$$

$$x = -4 : \quad 1 = A$$

$$x = -5 : \quad 1 = -B \Rightarrow B = -1$$

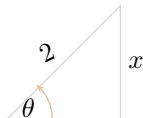
$$\frac{1}{(x+4)(x+5)} = \frac{1}{x+4} - \frac{1}{x+5}$$

$$\begin{aligned} & \int \frac{1}{(x+4)(x+5)} dx \\ &= \int \left(\frac{1}{x+4} - \frac{1}{x+5} \right) dx \\ &= \ln|x+4| - \ln|x+5| + C \\ &= \ln \left| \frac{x+4}{x+5} \right| + C. \end{aligned}$$



Solution to 8. Let

$$\begin{aligned}x &= 2 \sin \theta, \\dx &= 2 \cos \theta \, d\theta \\ \sqrt{4-x^2} &= 2 \cos \theta.\end{aligned}$$



$$\begin{aligned}& \int \frac{1}{x^2 \sqrt{1-x^2}} \, dx \\&= \int \frac{1}{4 \sin^2 \theta \, 2 \cos \theta} \, 2 \cos \theta \, d\theta \\&= \int \frac{1}{4} \csc^2 \theta \, d\theta \\&= \frac{-\cot \theta}{4} + C \\&= -\frac{\sqrt{4-x^2}}{4x} + C.\end{aligned}$$



Solution to 9.

$$\begin{aligned}8 - x^2 - 2x &= 8 - [x^2 + 2x], \\&= 8 - [x^2 + 2x + \textcolor{red}{1} - \textcolor{red}{1}] \\&= 8 - [(x + 1)^2 - 1] \\&= 8 - (x + 1)^2 + 1 \\&= 9 - (x + 1)^2.\end{aligned}$$

$$\begin{aligned}&\int \frac{1}{\sqrt{8 - x^2 - 2x}} dx \\&= \int \frac{1}{\sqrt{9 - (x + 1)^2}} dx \quad \int \frac{1}{\sqrt{a^2 - [f(x)]^2}} f'(x) dx = \sin^{-1} \left(\frac{f(x)}{a} \right) + C \\&= \sin^{-1} \left(\frac{x + 1}{3} \right) + C.\end{aligned}$$



Solution to 10.

$$\frac{1}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$$

$$1 = Ax(x+3) + B(x+3) + Cx^2$$

$$x = 0 : \quad 1 = 3B \Rightarrow B = \frac{1}{3}$$

$$x = -3 : \quad 1 = 9C \Rightarrow C = \frac{1}{9}$$

$$x = 1 : \quad 1 = 4A + 4B + C \Rightarrow 1 = 4A + \frac{4}{3} + \frac{1}{9}$$

$$1 = 4A + \frac{13}{9} \Rightarrow 4A = \frac{-4}{9}$$

$$A = \frac{-1}{9}$$

$$\frac{1}{x^2(x+3)} = \frac{\frac{-1}{9}}{x} + \frac{\frac{1}{3}}{x^2} + \frac{\frac{1}{9}}{x+3}$$

$$\begin{aligned}& \int \frac{1}{x^2(x+3)} dx \\&= \int \left(\frac{-1}{9x} + \frac{1}{9(x+3)} + \frac{1}{3x^2} \right) dx \\&= \frac{-1}{9} \ln|x| + \frac{1}{9} \ln|x+3| - \frac{1}{3x} + C \\&= \frac{1}{9} \ln \left| \frac{x+3}{x} \right| - \frac{1}{3x} + C.\end{aligned}$$



Solution to 11.

$$\begin{aligned}& \int \frac{1 + \sin x}{\sin^2 x} dx \\&= \int \left[\frac{1}{\sin^2 x} + \frac{\sin x}{\sin^2 x} \right] dx \\&= \int [\csc^2 x + \csc x] dx \\&= -\cot x + \ln |\csc x - \cot x| + C.\end{aligned}$$



Solution to 12. Let

$$w = \sqrt{x},$$

$$w^2 = x,$$

$$2w dw = dx$$

$$x^3 = w^6.$$

$$\begin{aligned} & \int \frac{\sqrt{x}}{1+x^3} dx \\ &= \int \frac{w}{1+w^6} 2w dw \\ &= 2 \int \frac{1}{1+(w^3)^2} w^2 dw \quad \int \frac{1}{a^2 + [f(x)]^2} f'(x) dx = \frac{1}{a} \tan^{-1} \left(\frac{f(x)}{a} \right) + C \\ &= \frac{2}{3} \int \frac{1}{1+(w^3)^2} 3w^2 dw \\ &= \frac{2}{3} \tan(w^3) + C \\ &= \frac{2}{3} \tan(x^{3/2}) + C. \end{aligned}$$

$$w = \sqrt{x} = x^{1/2}$$



Solution to 13.

$$\begin{aligned}& \int \frac{e^{2x}}{1+e^x} dx && u = e^x \, du = e^x \, dx \\&= \int \frac{e^x}{1+e^x} e^x dx \\&= \int \frac{u}{u+1} du \\&= \int \frac{u+1-1}{u+1} du \\&= \int \left[\frac{u+1}{u+1} - \frac{1}{u+1} \right] du \\&= \int \left[1 - \frac{1}{u+1} \right] du \\&= u - \ln |u+1| + C && u = e^x \\&= e^x - \ln(e^x + 1) + C.\end{aligned}$$



Solution to 14.

$$\begin{aligned}& \int_1^{\infty} \frac{8}{x^5} dx \\&= \lim_{t \rightarrow \infty} \int_1^t 8x^{-5} dx \\&= \lim_{t \rightarrow \infty} \left[\frac{8x^{-4}}{-4} \right]_1^t \\&= \lim_{t \rightarrow \infty} \left[\frac{-2}{x^4} \right]_1^t \\&= \lim_{t \rightarrow \infty} \left[\frac{-2}{t^4} + 2 \right] \\&= [0 + 2] = 2.\end{aligned}$$



Solution to 15.

$$\begin{aligned}x^2 + 2x + 5 &= (x^2 + 2x) + 5, \\&= (x^2 + 2x + \textcolor{red}{1} - \textcolor{red}{1}) + 5 \\&= (x + 1)^2 - 1 + 5 \\&= (x + 1)^2 + 4 = (x + 1)^2 + 2^2.\end{aligned}$$

$$\begin{aligned}&\int \frac{1}{\sqrt{x^2 + 2x + 5}} dx \\&= \int \frac{1}{\sqrt{(x + 1)^2 + 2^2}} dx \\&= \ln \left(x + 1 + \sqrt{(x + 1)^2 + 2^2} \right) + C \\&= \ln \left(x + 1 + \sqrt{x^2 + 2x + 5} \right) + C.\end{aligned}$$
$$\int \frac{f'(x)}{\sqrt{[f(x)]^2 + a^2}} dx = \ln |f(x) + \sqrt{[f(x)]^2 + a^2}| + C$$



Solution to 16.

$$x^2 - 7x + 12 = (x - 4)(x - 3),$$

$$\frac{x + 1}{(x - 4)(x - 3)} = \frac{A}{x - 4} + \frac{B}{x - 3}$$

$$x + 1 = A(x - 3) + B(x - 4)$$

$$x = 3 : \quad 4 = -B \Rightarrow B = -4$$

$$x = 4 : \quad 5 = A$$

$$\frac{x + 1}{(x - 4)(x - 3)} = \frac{4}{x - 4} + \frac{-5}{x - 3}$$

$$\begin{aligned} \int \frac{x + 1}{\sqrt{x^2 - 7x + 12}} dx &= \int \left(\frac{5}{x - 4} + \frac{-4}{x - 3} \right) dx \\ &= 5 \ln |x - 4| - 4 \ln |x - 3| + C \\ &= \ln |x - 4|^4 - \ln (x - 3)^4 + C \\ &= \ln \left(\frac{|x - 4|^5}{(x - 3)^4} \right) + C. \end{aligned}$$



Solution to 17.

$$\begin{array}{r}
 x \\
 x^2 - 1 \overline{) \begin{array}{l} x^3 + x \\ -x^3 + x \\ \hline 2x \end{array}}
 \end{array}$$

$$\begin{aligned}
 \frac{x^3 + x}{x^2 - 1} &= x + \frac{2x}{x^2 - 1} \\
 \int \frac{x^3 + x}{x^2 - 1} dx &= \int \left(x + \frac{2x}{x^2 - 1} \right) dx \\
 &= \frac{1}{2}x^2 + \ln |x^2 - 1| + C.
 \end{aligned}$$



Solution to 18.

$$\frac{4x+6}{(x^2+1)(x-1)^2} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$$

$$4x+6 = (Ax+B)(x-1)^2 + C(x-1)(x^2+1) + D(x^2+1)$$

$$4x+6 = (Ax+B)(x^2-2x+1) + C(x^3-x^2+x-1) + D(x^2+1)$$

$$4x+6 = (A+C)x^3 + (-2A+B-C+D)x^2 + (A-2B+C)x + (B-C+D)$$

$$x=1: \quad 10 = 2D \Rightarrow D = 5.$$

Also, we have

$$\text{coeff.}x^3: \quad 0 = A + C \quad (1)$$

$$\text{coeff.}x^2: \quad 0 = -2A + B - C + D \quad (2)$$

$$\text{coeff.}x: \quad 4 = A - 2B + C \quad (3)$$

$$\text{constant}: \quad 6 = B - C + D \quad (4)$$

$$(5)$$

Using (4) and (2) we get that $0 = -2A + 6 \Rightarrow A = 3$.

Using (1) we get $C = -A = -3$.

Using (3) we get $4 = 3 - 2B - 3 \Rightarrow 4 = -2B \Rightarrow B = -2$.
Therefore

$$\frac{4x + 6}{(x^2 + 1)(x - 1)^2} = \frac{3x - 2}{x^2 + 1} + \frac{-3}{x - 1} + \frac{5}{(x - 1)^2}$$



Solution to 19.

$$\begin{aligned}\int_0^{\infty} \frac{1}{e^x + e^{-x}} dx &= \lim_{t \rightarrow \infty} \int_0^t \frac{1}{e^x + e^{-x}} dx \\&= \lim_{t \rightarrow \infty} \int_0^t \frac{1}{e^x + e^{-x}} \frac{e^x}{e^x} dx \\&= \lim_{t \rightarrow \infty} \int_0^t \frac{1}{e^{2x} + 1} e^x dx \\&= \lim_{t \rightarrow \infty} \int_0^t \frac{1}{(e^x)^2 + 1} e^x dx \\&= \lim_{t \rightarrow \infty} [\tan(e^x)]_0^t \\&= \lim_{t \rightarrow \infty} [\tan(e^t) - \tan 1] = \left[\frac{\pi}{2} - \frac{\pi}{4} \right] = \frac{\pi}{2}.\end{aligned}$$



Solution to 20. Note that

$$\lim_{t \rightarrow 0^+} t^2 \ln t \quad (0.(-\infty)) \quad \text{I.F.}$$

$$\lim_{t \rightarrow 0^+} t^2 \ln t = \lim_{t \rightarrow 0^+} \frac{\ln t}{\frac{1}{t^2}} \quad \left(\frac{-\infty}{\infty} \right) \quad \text{I.F.}$$

$$\stackrel{L.H.}{=} \lim_{t \rightarrow 0^+} \frac{\frac{1}{t}}{\frac{-2}{t^3}}$$

$$= \lim_{t \rightarrow 0^+} \frac{t^2}{-2}$$

$$= 0.$$

$$\begin{aligned}\int_0^1 x \ln x \, dx &= \lim_{t \rightarrow 0^+} \int_t^1 x \ln x \, dx \\&= \lim_{t \rightarrow 0^+} \int_t^1 \ln x \, d\left(\frac{1}{2}x^2\right) && u = \ln x \quad dv = x \, dx \\&= \lim_{t \rightarrow 0^+} \left[\frac{1}{2}x^2 \ln x \Big|_t^1 - \int_t^1 \frac{1}{2}x^2 \frac{1}{x} \, dx \right] && du = \frac{dx}{x} \quad v = \frac{1}{2}x^2 \\&= \lim_{t \rightarrow 0^+} \left[0 - \frac{1}{2}t^2 \ln t - \left[\frac{1}{4}x^2\right]_t^1 \right] \\&= \lim_{t \rightarrow 0^+} \left[-\frac{1}{2}t^2 \ln t - \frac{1}{4} + \frac{1}{4}t \right] \\&= \left[0 - \frac{1}{4} \right] = \frac{-1}{4}.\end{aligned}$$



Solution to 21.

$$\begin{aligned}& \int_1^e \frac{1}{x\sqrt[3]{\ln x}} dx \\&= \lim_{t \rightarrow 1^+} \int_t^e (\ln x)^{-1/3} \frac{1}{x} dx \\&= \lim_{t \rightarrow 1^+} \left[\frac{3}{2} (\ln x)^{2/3} \right]_t^e \\&= \lim_{t \rightarrow 1^+} \left[\frac{3}{2} (\ln e)^{2/3} - \frac{3}{2} (\ln t)^{2/3} \right] = \frac{3}{2}.\end{aligned}$$



Solution to 22.

$$\frac{3x^2 - x + 5}{x(x^2 + 5)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 5}$$

$$3x^2 - x + 5 = A(x^2 + 5) + (Bx + C)x$$

$$3x^2 - x + 5 = Ax^2 + 5A + Bx^2 + Cx$$

$$3x^2 - x + 5 = (A + B)x^2 + Cx + 5A$$

We have

$$\text{coeff. } x^2 : \quad 3 = A + B \quad \text{--- (1)} \quad (6)$$

$$\text{coeff. } x : \quad -1 = C \quad \text{--- (2)} \quad (7)$$

$$\text{constant :} \quad 5 = 5A \quad \text{--- (3)} \quad (8)$$

$$(9)$$

Using (3) we get that $A = 1$.

Using (1) we get $B = 3 - A = 3 - 1 = 2$. Using (2) we get $C = -1$.

$$\text{Therefore } \frac{3x^2 - x + 5}{x(x^2 + 5)} = \frac{1}{x} + \frac{2x - 1}{x^2 + 5} = \frac{1}{x} + \frac{2x}{x^2 + 5} - \frac{1}{x^2 + 5}$$

$$\begin{aligned} & \int \frac{3x^2 - x + 5}{x(x^2 + 5)} dx \\ &= \int \left(\frac{1}{x} + \frac{2x}{x^2 + 5} - \frac{1}{x^2 + 5} \right) dx \\ &= \ln|x| + \ln|x^2 + 5| - \frac{1}{\sqrt{5}} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + C \\ &= \ln|x(x^2 + 5)| - \frac{1}{\sqrt{5}} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + C \\ &= \ln|x^3 + 5x| - \frac{1}{\sqrt{5}} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + C. \end{aligned}$$



Solution to 23. Let

$$w = \sqrt[3]{x},$$

$$w^3 = x,$$

$$3w^2 dw = dx$$

$$\int \frac{1}{x + \sqrt[3]{x}} dx$$

$$= \int \frac{1}{w^3 + w} 3w^2 dw$$

$$= \frac{3}{2} \int \frac{2w}{w^2 + 1} dw \quad \int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$$

$$= \frac{3}{2} \ln |w^2 + 1| + C \quad w = \sqrt[3]{x}$$

$$= \frac{3}{2} \ln |x^{2/3} + 1| + C.$$



Solution to 24.

$$\begin{aligned}& \int \tan x \sec^3 x \, dx \\&= \int \sec^2 x \sec x \tan x \, dx \quad u = \sec x \quad du = \sec x \tan x \, dx \\&= \int u^2 \, du \\&= \frac{u^3}{3} + C \quad u = \sec x \\&= \frac{\sec^3 x}{3} + C.\end{aligned}$$



Solution to 25.

Using the fact

$$\begin{aligned}\int \frac{1}{\sqrt{x^2 + a^2}} dx &= \ln \left| \frac{x}{a} + \frac{\sqrt{x^2 + a^2}}{a} \right| + C \\ &= \ln |x + \sqrt{x^2 + a^2}| + C\end{aligned}$$

$$\int \frac{1}{\sqrt{x^2 + 4}} dx = \ln |x + \sqrt{x^2 + 4}| + C = \ln \left| \frac{x}{2} + \frac{\sqrt{x^2 + 4}}{2} \right| + C.$$

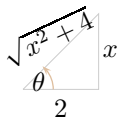
Another solution: Let

$$x = 2 \tan \theta,$$

$$dx = 2 \sec^2 \theta d\theta$$

$$\sqrt{x^2 + 4} = 2 \sec \theta.$$

$$\begin{aligned} & \int \frac{1}{\sqrt{x^2 + 4}} dx \\ &= \int \frac{1}{2 \sec \theta} 2 \sec^2 \theta d\theta \\ &= \int \sec \theta d\theta \\ &= \ln |\sec \theta + \tan \theta| + C \\ &= \ln \left| \frac{x}{2} + \frac{\sqrt{x^2 + 4}}{2} \right| + C. \end{aligned}$$



Solution to 26.

$$\begin{aligned}& \int \frac{x^3 + 8}{x + 2} dx \\&= \int \frac{x^3 + 8}{x + 2} dx & x^3 + 8 = x^3 + 2^3 = (x + 2)(x^2 - 2x + 4) \\&= \int \frac{(x + 2)(x^2 - 2x + 4)}{x + 2} dx \\&= \int (x^2 - 2x + 4) dx \\&= \frac{x^3}{3} - x^2 + 4x + C.\end{aligned}$$



Solution to 27.

$$\begin{aligned}& \int (\cos^4 x - \sin^4 x) dx \\&= \int (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) dx && \cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) \\&= \int (\cos^2 x - \sin^2 x) dx && \cos^2 x + \sin^2 x = 1 \\&= \int (\sin x + \cos x)(\cos x - \sin x) dx && \cos^2 x - \sin^2 x = (\cos x - \sin x)(\cos x + \sin x) \\&= \int (\sin x + \cos x)(\cos x - \sin x) dx && d(\cos x + \sin x) = (\cos x - \sin x) dx \\&= \frac{1}{2}(\sin x + \cos x)^2 + C_1 \\&= \frac{1}{2}[\cos^2 x + 2 \sin x \cos x + \sin^2 x] + C_1 && \cos^2 x + \sin^2 x = 1 \\&= \frac{1}{2}[1 + 2 \sin x \cos x] + C_1 && \frac{1}{2} + C_1 = C \\&= \sin x \cos x + C.\end{aligned}$$



Solution to 28. Let

$$w = \sqrt{x+1},$$

$$w^2 = x+1,$$

$$2w dw = dx$$

$$w^2 - 1 = x$$

$$\int x\sqrt{x+1} dx$$

$$= \int (w^2 - 1)w 2w dw$$

$$= \int (2w^4 - 2w^2) dw$$

$$= \frac{2}{5}w^5 - \frac{2}{3}w^3 + C$$

$$w = \sqrt{x+1}$$

$$= \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C.$$



Solution to 29. Let

$$w = e^x,$$

$$dw = e^x dx$$

$$\int \frac{e^x}{(e^x + 2)(e^x + 3)} dx = \int \frac{1}{(w + 2)(w + 3)} dw$$

$$\frac{1}{(w + 2)(w + 3)} = \frac{A}{w + 2} + \frac{B}{w + 3}$$

$$1 = A(w + 3) + B(w + 2)$$

$$w = -2 : \quad 1 = A$$

$$w = -3 : \quad 1 = -B \Rightarrow B = -1$$

$$\frac{1}{(w + 2)(w + 3)} = \frac{1}{w + 2} - \frac{1}{w + 3}$$

$$\begin{aligned}& \int \frac{e^x}{(e^x + 2)(e^x + 3)} dx \\&= \int \frac{1}{(w + 2)(w + 3)} dw \\&= \int \left[\frac{1}{w + 2} - \frac{1}{w + 3} \right] dw \\&= \ln |w + 2| - \ln |w + 3| + C \quad w = e^x \\&= \ln (e^x + 2) - \ln (e^x + 3) + C.\end{aligned}$$



Solution to 30.

$$\begin{aligned}\int_1^{\infty} e^{-x^2} 2x dx &= - \lim_{t \rightarrow \infty} \int_1^t e^{-x^2} - 2x dx \\&= \lim_{t \rightarrow \infty} \left[-e^{-x^2} \right]_1^t \\&= \lim_{t \rightarrow \infty} \left[-e^{-t^2} + e^{-1} \right] = 0 + e^{-1} = \frac{1}{e}.\end{aligned}$$



Solution to 31.

$$\begin{aligned}& \int \sin (5) \cos (7 x) d x \\&= \sin (5) \int \cos (7 x) d x \quad \int \cos (a x)=\frac{1}{a} \sin (a x)+C \\&= \sin (5) \frac{1}{7} \sin (7 x)+C \\&= \frac{\sin (5) \sin (7 x)}{7}+C.\end{aligned}$$

